BAT (BAT ALGORITHM) –BASED FUZZY PD CONTROL DESIGN FOR THE STABILIZATION OF A QUADROTOR

Sarra Bencharef and Hamid Boubertakh

Department of Automatic, Automatic Laboratory of Jijel (Laj) University

Msby of Jijel, Algeria

ABSTRACT

In this work, we propose a new design approach based on bat algorithm for optimal design of Fuzzy-Proportional-Derivative controllers (FPD) for the stabilization of a Quadrotor. To stabilize the angles (roll, pitch and yaw) and heights of the Quadrotor a decentralized control structure is adopted where four FPD controllers are used. Their parameters are given simultaneously by BAT algorithm. The performance of the system from its desired behavior is quantified by an objective function (SE). Some simulation results are presented to show the efficiency of the method .

KEYWORDS

Bat algorithm (BA), Fuzzy PD, Quadrotor, Optimization, Control

1. INTRODUCTION

Today unmanned aerial vehicle (UAVs) are a big hit on the side of the automatic control researchers, for their various applications in civilian and military tasks. The control objective of a Quadrotor can be the angles stabilization, the height stabilization, the angles and height stabilization, or position stabilization. Many control techniques are applied to achieve these objectives; PID control, optimal Control, backsteeping control, and sliding mode control [1-3].

The PID control is relevant, due to the simplicity of this structure and its ease of implementation. However, it has limited performance because of its structure and the linear system and an opposite non-linear and complex system. In addition, an incorrect setting of these parameters will lead to a more performance degradation. Unfortunately, there are no systematic methods for setting parameters and the existing methods do not guarantee optimal operation such as Ziegler Nichols method [4-5]. Fuzzy PID control [6] is presented in literature as an alternative to address the difficulties with classical PID control. It combines the advantages these controllers namely the simplicity, the interpretability of the control structure and more they can deal with non linear systems.

In recent years, many heuristic evolutionary optimization algorithms have been developed. These include Genetic Algorithm (GA) [7-8], Ant Colony optimization (ACO) [9-10], Particle Swarm Optimization (PSO) [11-12], and Gravitational Search algorithm (GSA) [12-13], Bat Algorithm (BA) [14-15] . In this paper, we suggest the use of Bat Algorithm in order to solve the Fuzzy PD control design problem for angles and height stabilization of a Quadrotor.

The remainder of this paper is organized as follows. Section II gives a general description and presents a dynamical model of a Quadrotor. Section III gives an Overview of the BA and presents the Fuzzy PD controller design method using BA. In section IV, first the Fuzzy PD decentralized control structure for the stabilization of the Quadrotor is presented then BA design method is used and simulation results are presented. Section V concludes the paper.

2. THE QUADROTOR MODEL

The Quadrotor is an UAV minimized with four propellers mounted on the end of two perpendicular axes and actuated by four DC motors. A basic configuration of Quadrotors is shown in Figure 1.

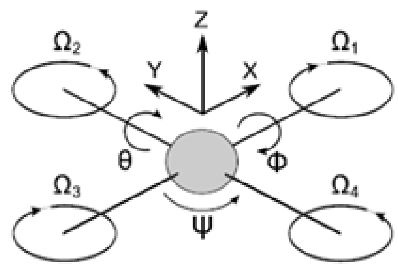

Figure 1. Quadrotor configuration

The four rotors which form two pairs rotate inversely, one clockwise and the other counter clockwise.We consider the dynamical state model of a Quadrotor in[2] given by:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_4 x_6 a_1 + a_2 x_4 \Omega_r + b_1 u_2 \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = x_2 x_6 a_3 - a_4 x_2 \Omega_r + b_2 u_3 \\ \dot{x}_5 = x_6 \\ \dot{x}_6 = x_4 x_2 a_5 + b_3 u_4 \\ \dot{x}_7 = x_8 \\ \dot{x}_8 = -g + (\cos x_1 * \cos x_3) b_4 u_1 \\ \dot{x}_9 = x_{10} \\ \dot{x}_{10} = u_x b_4 u_1 \\ \dot{x}_{11} = x_{12} \\ \dot{x}_{12} = u_y b_4 u_1 \end{cases} \quad (1)$$

Where:

- $x_1 = \varphi$ and $x_2 = \dot{\varphi}$ are respectively the roll angle and corresponding angular velocity;
- $x_3 = \theta$ and $x_4 = \dot{\theta}$ are respectively the pitch angle and corresponding angular velocity;
- $x_5 = \psi$ and $x_6 = \dot{\psi}$ are respectively the yaw angle and corresponding angular velocity;
- $x_7 = Z$, $x_9 = X$, and $x_{11} = Y$ are the cartesian position coordinates.
- $x_8 = \dot{Z}$, $x_{10} = \dot{X}$, and $x_{12} = \dot{Y}$ are translation velocities

- $u_x = cosx_1 * sinx_3 * cosx_5 + sinx_1 * sinx_5$
- $u_x = cosx_1 * sinx_3 * sinx_5 - sinx_1 * cosx_5$
- $\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} b & b & b & b \\ 0 & -b & 0 & b \\ b & 0 & -b & 0 \\ -d & d & -d & d \end{bmatrix} \begin{bmatrix} w_1^2 \\ w_2^2 \\ w_3^2 \\ w_4^2 \end{bmatrix}$
- $\Omega_r = -w_1 + w_2 - w_3 + w_4$ is the sum of rotor angular velocities
- $a_1 = \frac{I_y - I_z}{I_x}; a_2 = \frac{J_r}{I_x}; a_3 = \frac{I_z - I_x}{I_y}; a_4 = \frac{J_r}{I_y} \quad ; a_5 = \frac{I_x - I_y}{I_z}, b_1 = \frac{l}{I_x}, \; b_2 = \frac{l}{I_y}; \; b_3 = \frac{1}{I_z}; b_4 = \frac{1}{m}$
- The constraints on the control inputs are governed by the bound on the motors' angular velocities i=1….4, which are $\Omega_r \in [0.\Omega_i^{max}]$. As a result the constraints on the control inputs are formulated as shown in (2).

$$u_1^{min} = 0 \le u_1 \le b\sum_{i=1}^{4}(\Omega_1^{max})^2 = u_1^{max}$$

$$u_2^{min} = -b(\Omega_2^{max})^2 \le u_2 \le -b(\Omega_4^{max})^2 = u_2^{max}$$

$$u_3^{min} = -b(\Omega_3^{max})^2 \le u_3 \le -b(\Omega_1^{max})^2 = u_3^{max} \qquad (2)$$

$$u_4^{min} = -d\left[(\Omega_1^{max})^2 + (\Omega_3^{max})^2\right] \le u_4 \le \; -b\left[(\Omega_2^{max})^2 + (\Omega_4^{max})^2\right] = u_4^{max}$$

$$\Omega_r^{min} = -\Omega_1^{max} - \Omega_3^{max} \le \Omega_r \le \Omega_2^{max} + \Omega_1^{max} = \Omega_r^{max}$$

The values of the parameters in the state space model are given by Table 1 [2].

Table 1. The Quadrotor model parameter

Parameter	Value
l	0.230 (m)
m	0.65(Kg)
d	$7.5e-7$ ($N.m.s2$)
b	3.13e-5(N. s^2)
I_X	7.5e-3 (Kg.m^2)
I_Y	7.5e-3 (Kg.m^2)
I_Z	1.3e-2(Kg.m^2)
I_R	6e-5(Kg.m^2)
g	9.806m / s2

3. THE PD CONTROL DESIGN USING BA

3.1. OVERVIEW OF THE BAT ALGORITHM

The Bat Algorithm (BA) is a bio- inspired algorithm developed by Xin-She Yang in 2010 [14]. The bat algorithm is based on the echolocation behaviour of micro bats with varying pulse emission and loudness. The invention of echolocation can be summarized as follows: Each virtual bat flies randomly with a velocity v_i at position $x_i$with a varying frequency at i^{tk}step. Search is strengthened by a local random walk. Selection of the best continues until certain stop criteria are met. Firstly, the initial position x_i of, velocity v_i and frequency f_i are initialized for each bat. For each step t, the movement of the virtual bat is given by updating their velocity and position using (3), (4) and (5) as follows.

$$f_i = f_{min} + (f_{min} - f_{max})\beta \quad (3)$$

$$v_i^j(t) = v_i^j(t-1) + [x_{cgbest}^j - X_i^j(t-1)]f_i \quad (4)$$

$$X_i^d(t+1) = X_i^d(t) + v_i^d(t+1) \quad (5)$$

The result of equation (3) is used to control the space and the range of bats mouvement

Where:

β is random generated number within the interval $[0,1]$
The variable x_{cgbest}^j represents the current global best solution for decision variable j , which is achieved comparing all the solutions provided by the *n* bats.

For the local search part, once a solution is selected among the current best solutions, a new solution for each bat is generated locally using random walk:

$$X_i^{new} = X_i^{old} + \sigma A_{mean}^{old} \quad (6)$$

Where:

σ is a random number
A_{mean}^{old} is the average loudness of all the bats at this time step.

3.2. THE FUZZY PD PARAMETERS TUNING PROBLEM USING BA

The fuzzy Proportional Derivative controller FPD is defined by the following equations

$$U_{FPD}(t) = f(e(t), \frac{de(t)}{dt}) \quad (7)$$

f is the fuzzy inference system function

The error $e(t)$ is defined as:

$$e(t) = y_r(t) - y(t) \quad (8)$$

u is the control signal, y_r is the reference signal and y is the controller output

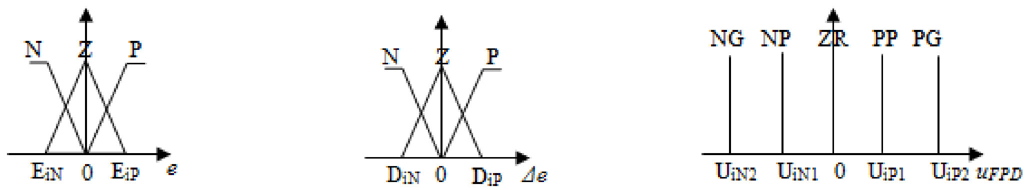

Figure 2. Membership functions of the FPD controllers[16]

According to the typical structural design of FPD controllers in [16], one uses FPD controllers with 3membership functions for input variables and 5 singletons for the output variable as shown on Fig. 2.The rule base of the controllers is donated by Table 2.

The crisp control action is determined by the average sum:

$$U_{PFD}(k) = \frac{\sum_{i=1}^{9} V_i C_i}{\sum_{i=1}^{9} V_i} \quad (9)$$

C_i is the conclusion of rule i and V_i is its truth value calculated by the Larsen's product method given by:

$$V_i = \mu_{e_i}(e(k)).\mu_{\Delta e_i}(\Delta e(k)) \quad (10)$$

$\mu_{yi}(y)$ is the membership grade of the input variable y evaluated in rule i by the corresponding membership function.

Table 2. The FPD controller's rule base [16]

e	Δe		
	N	Z	P
N	c_1=NB	c_2=NS	c_3=ZR
Z	c_4=NS	c_5=ZR	c_6=PS
P	c_7=ZR	c_8=PS	c_9=PB

The labels NB, NS, ZR, PS, and PB refer respectively to the linguistic terms; Negative Big, Negative Small, around Zero, Positive Small and Positive Big.

Note here that if we consider that the input membership functions and output singletons are evenly distributed on symmetrical universes of discourse (i.e.$\boldsymbol{E_{iN} = E_{iP}}$, $\boldsymbol{D_{iN} = D_{iP}}$and $\boldsymbol{U_{iN2} = U_{iP2} = 2U_{iN1} = 2U_{iP1} = U}$).

A nonlinear FPD controller can be reached by moving the input membership functions and/or the output singletons from their modal positions. That is, the FPD controllers combine the advantages of classical PD controllers and fuzzy controllers; the simplicity, the structure interpretability, and the nonlinearity.

The design parameters' vector is constructed depending on the control objective and the available knowledge about the system under control; the positions of the input membership functions and/or the positions of the output singletons. In the general case, the design parameters' vector can be expressed by:

$$\boldsymbol{P = [E_{iN}\ \ E_{iP}\ D_{iN}\ D_{iP}\ \ U_{iN1}\ U_{iN2}\ U_{iP1}\ U_{iP2}]} \tag{11}$$

Without loss of generality, a squared error cost function (12) is used to quantify the effectiveness of a given FPD controllers; it is evaluated at the end of desired input signal of the closed-loop system under control.

$$\boldsymbol{ISE = \int_{t_0}^{t_f} e(t)^2 dt} \tag{12}$$

In the PD controller design problem using GSA, an agent is one value of the PD controller parameters vector.

4. APPLICATION TO THE STABILIZATION OF A QUADROTOR

The control objective is to stabilize angles and height of the Quadrotor. That is, we use four decentralized PD controllers as presented by the bloc diagram on the Figure 3.

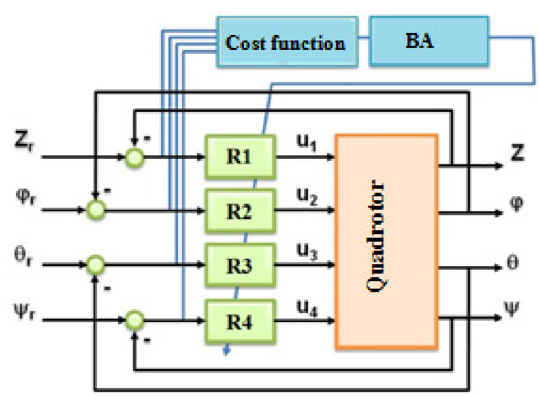

Figure 3. Decentralized PD control structure

Because of Quadrotor body symmetry, we take identical controllers for the roll and pitch angles **(i.e. PD_1 =PD_2)** The design parameters' vector is composed of the parameters of the PD controllers:

$$FPD = [E_1\ D_2 U_{11} U_{12} E_3\ D_3 U_{31} U_{32} E_4\ D_4 U_{41} U_{42}]^T \tag{13}$$

The effectiveness of the FPD controllers is evaluated at the end of the response of the entire control system to desired input signals by the squared error fitness function given by:

$$ISE = \sum_{i=1}^{4} \int_{t_0}^{t_f} e(t)^2 dt \tag{14}$$

In the simulations, we take:

- The constraints on the inputs have been computed based on the physical parameters of the system and specifically:
  a) The maximum angular velocity of the motors (assuming the characteristics of a dc motor as provided in [3]),
  b) The thrust factor in hovering, computed $b = 2.8 * 10^{-5}\ N/sec^2$ and c) the drag factor in hovering, computed as $d = 8 * 10^{-7}\ \text{m}/sec^2$. Based on the above values the following constraints on the inputs can be set as $0 \le U1 \le 11.23$, $|U2| \le 5.61$, $|U3| \le 5.61$, $|U4| \le 0.16$. [14].
- The initial states : $\varphi(0) = 1\ rad$, $\theta(0) = -1\ rad$, $\psi(0) = 1\ rad, Z(0) = 0\ m.$
- The desired angles and height: $\varphi_d = 0\ rad$, $\theta_d = 0 rad$, $\psi_d = 0\ rad$ and $Z_d = 5\ m.$
- The cost function spans from $t_0 = 0\ s$, $t_f = 25\ s$
- The initial parameters values are chosen randomly between the bounds given by Table 3.
- For the BAT algorithm (BA) we take the parameters by Table 4.

Table 3. The initial parameters bounds for Fuzzy PD controller

Parameter	Min	Max
E_i, i = 1: 4	0	1
D_I, i = 1: 4	0	1
U_{i1}^i, i = 1: 4	0	0.5
U_{i2}^i, i = 1: 4	0.5	1

Table 4. The bat algorithm parameters

Parameter	**value**
Population size n	50
Frequency minimum f_{min}	0
Frequency maximum f_{max}	1
Loudness A	0.9
Pulse rate r	0.2
Number of generation K	100

5. Simulation Results And Discussion

The final parameters values are given on Table 5.The Figure 4 show the evolution of the cost function with iterations. Figures 5, 6,7 and 8 shows respectively the roll angle, the pitch angle, the yaw angle and the height with the tuned controllers.

Table 5. The final tuned parameters values

Regulator	**Parameter**	**Values optimal**
$\boldsymbol{FPD_4}$	E_4 D_4 U_4 U_{44}	0.8787 0.6187 8.1667 99.8014
$\boldsymbol{FPD_3}$	E_3 D_3 U_3 U_{33}	34.7130 41.0524 36.8041 40.2621
$\boldsymbol{FPD_2}$	E_2 D_2 U_2 U_{22}	14.4784 40.6856 28.2419 42.6087
$\boldsymbol{FPD_1}$	E_1 D_1 U_1 U_{11}	14.4784 40.6856 28.2419 42.6087

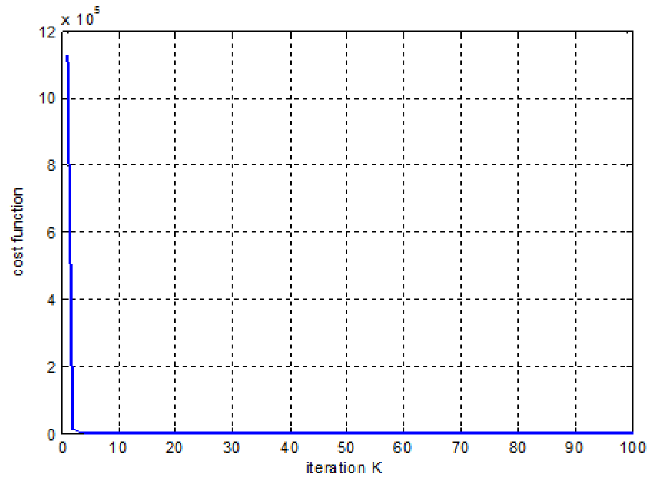

Figure 4. The Cost function

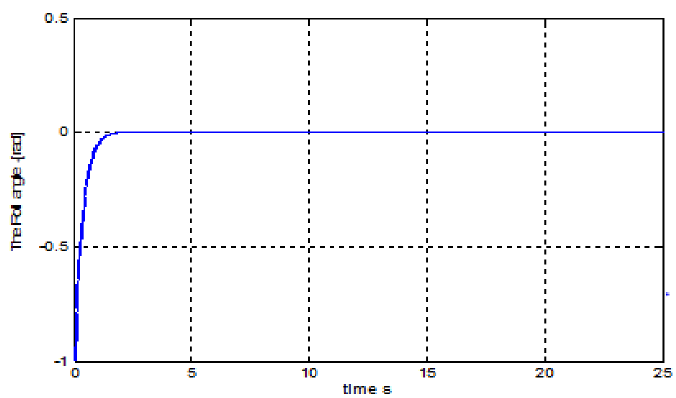

Figure 5. The Roll angle φ

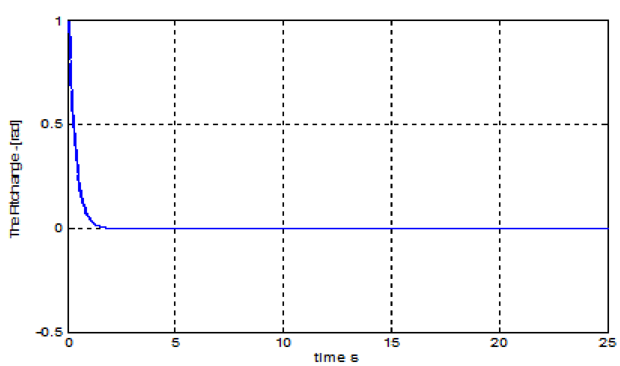

Figure 6. The Pitch angle θ

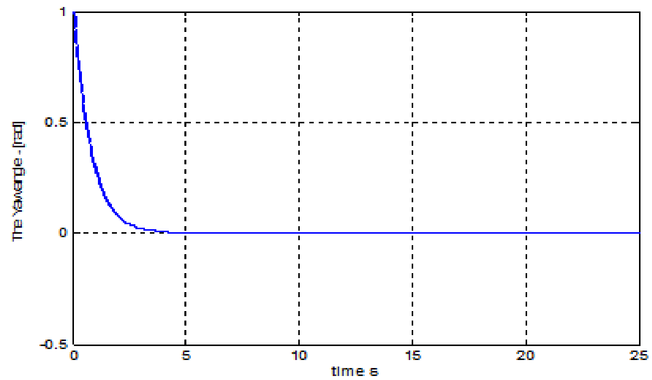

Figure 7. The Yaw angle ψ

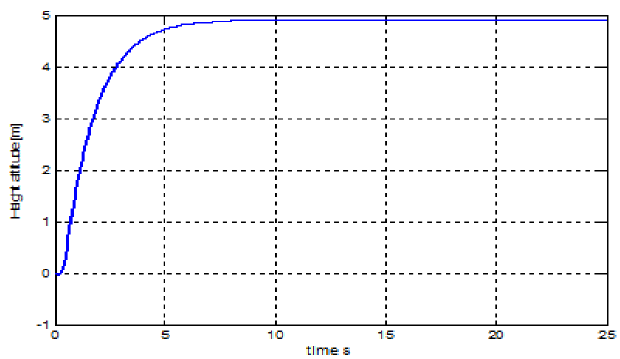

Figure 8. The Height Z

From these figures, we can see well a good tracking of the desired values of altitude and desired angles for the algorithms BAT Algorithm (BA) , (illustrated respectively by Fig. 5.6 .7 and 8). After the results we can see that the proposed algorithm give us satisfactory results because it provide good stability of Quadrotor. We can simply say that Bat Algorithm has shown its effectiveness in terms of stability Quadrotor, and can be added to the list of smart methods to ensure the stability of Quadrotor.

6. Conclusion And Future Work

A heuristic evolutionary optimization algorithm was proposed for the stabilization of a Quadrotor under the actuators saturation. This algorithm BA (BAT algorithm) is used to tune the FPD controllers' parameters simultaneously regarding a sum of square error cost function. The result of the simulation was able to yield results that showed good stability of the Quadrotor. The simulation results show the efficiency of the proposed method.Future study would be to study a stability of Quadrotor in the presence of external disturbances.

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AUTHORS

Sarra Bencharef is currently a Ph.D. student at the Automatic Control Department of Jijel University, Algeria. She received the Engineer degree in Electronic from Jijel University, Algeria, in 2009, and the Magister degree of Automatic Control and signal processing at the Automatic Control Department of Jijel University, Algeria, in April 2014. She is with Automatic Laboratory of Jijel (LAJ). His research interests include Aerial robotics, Linear and Nonlinear control, intelligent control and optimization.

Hamid Boubertakh graduated from National Polytechnic school of Algiers (ENP) in Algérie in 2009. He currently works as a Teacher-Researcher at the University of Jijel, Algeria, where he continues his research activities .His areas of interest are intelligent control and optimization.